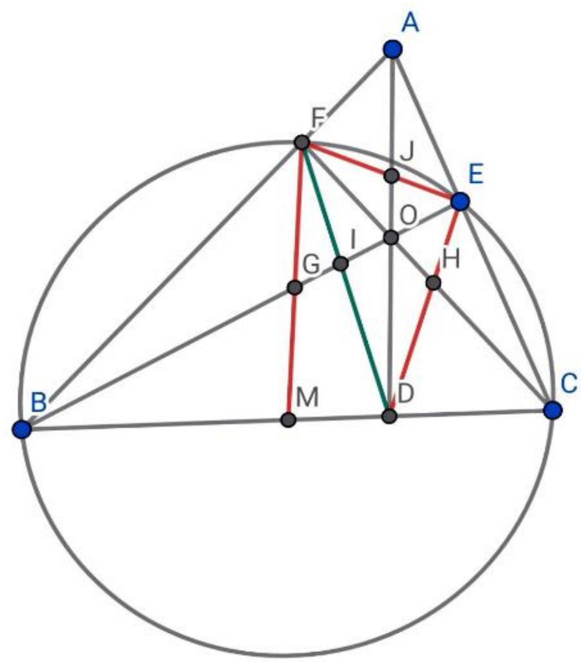
Author's Solution -Oct 2025



Construction :

Join DF. Let BE intersect FM & FD at G & I respectively. Let CF intersect DE at H. Let AD intersect FE at J.

By Concurrency Theorem (The Geometry of concurrency by Dr. M. Rajaclimax)

$\frac{AJ}{JO} = \frac{AD}{DO}, \frac{BI}{IO} = \frac{BE}{EO}, \frac{CH}{HO} = \frac{CF}{FO}$ ------------ **(1)**

Given BCEF concyclic, BE & CF intersect at O.

OE x OB = OF x OC

ie $\frac{OF}{OE} = \frac{OB}{OC}$ --------------------------**(2)**

In $\Delta BCO, MF\ is\ transvesal. By\ Menlaus\ Theorem$

$\frac{BM}{MC} \times \frac{CF}{FO} \times \frac{OG}{GB} = 1$ **(**$\because BM = MC,\ M$ **midpoint)**

$\frac{CF}{OF} = \frac{GB}{OG}$ ------------------------------ **(3)**

From (1) & (3)

$$\frac{CF}{OF} = \frac{GB}{OG} = \frac{CH}{OH}$$

$$\frac{GB}{OG} = \frac{CH}{OH}$$

$$\frac{GB + OG}{OG} = \frac{CH + OH}{OH}$$

$$\frac{OB}{OG} = \frac{OC}{OH}$$

$\frac{OB}{OC} = \frac{OG}{OH}$ ---------------------------- **(4)**

From (2) & (4)

$$\frac{OF}{OE} = \frac{OB}{OC} = \frac{OG}{OH}$$

$\Longrightarrow$ OF x OH = OE x OG.

$\Longrightarrow$ Chord FH & GE intersect at O

$\Longrightarrow$ GHEF is concyclic

$\Longrightarrow \angle FGE = \angle FHE$

$= \angle DHC$ (vertically opp. angle) ----------------- (5)

Consider $\Delta GEF \ \& \ \Delta DHC.$

As FECB is concyclic

$\angle FEB = \angle FCD$

$\Longrightarrow \angle FEG = \angle HCD$ ---------------------------(6)

$Also \angle FGE = \angle DHC$ [by (5)]

$\therefore \Delta HDC \sim \Delta GFE$ (by AA similarity)

$\Longrightarrow \angle GFE = \angle HDC$

$\Longrightarrow \angle MFE = \angle EDC$ (Exterior angle = opp. interior angle)

$\Longrightarrow$ MDEF is concyclic. ----------------------- Proved

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